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CAPABILITY ESTIMATE FOR A TRANSIT TRAFFIC SIGNAL SYSTEM

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ABSTRACT:
Traffic-signal control systems coordinate individual traffic signals to achieve network-wide traffic operations objectives. These systems consist of intersection traffic signals, a communications network to tie them together, and a central computer or network of computers to manage the system. Coordination can be implemented through a number of techniques including time-base and hardwired interconnection methods. Coordination of traffic signals across agencies requires the development of data sharing and traffic signal control agreements. Therefore, a critical institutional component of Traffic Signal Control is the establishment of formal or informal arrangements to share traffic control information as well as actual control of traffic signal operation across jurisdictions. Signal coordination systems are installed to provide access. A traffic-signal system has no other purpose than to deliver favorable signal timings to motorists. The system provides features that improve the traffic engineer's ability to achieve this goal. These are primarily access features. They provide access to the intersection signal controller for maintenance and operations. The more complete and convenient the access, the more efficient the operator will be and the more effective the system. In this paper, This traffic signal system is based on three colour signals, namely red, green and yellow. The red colour signal is used to stop the traffic and the green colour signal is used to moving of stopped traffic. The yellow colour signal indicates ready to move and it works prior to green signal.

KEY WORDS : traffic signal system, environmental failure

INTRODUCTION:

Since, the system under consideration is of Non-Markovian nature, the supplementary variable technique has been used to mathematical formulation of system. State-transition diagram has shown in fig -1. Difference-differential equations for all the transition-states have been obtained. Laplace transform has been used to solve these mathematical equations. Availability and profit function of considered system have computed. Ergodic behaviour of the system and a particular case (when repairs follow exponential time distribution) have been obtained to improve practical utility of the model. A numerical example together with its graphical illustration has been also appended in the end to highlight important results of the study.

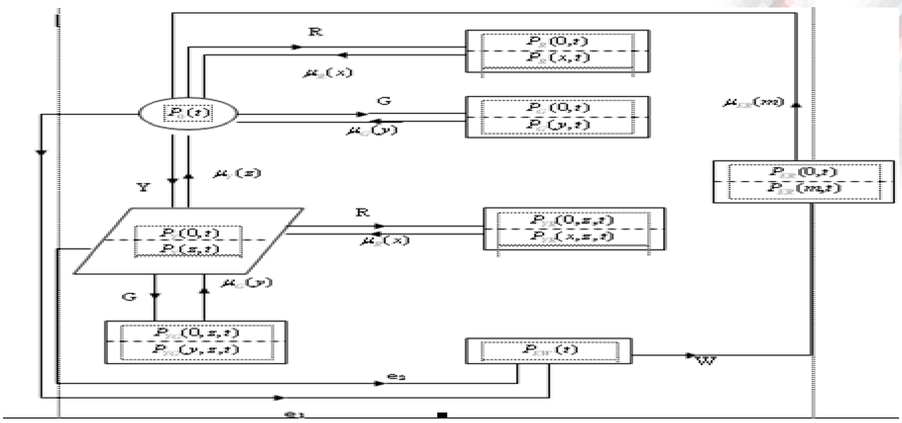

Fig-1 (State-Transition Diagram

NOTATIONS USED:

The following notations have been used throughout this model:

$P_0(t)$	Pr {at time t, system is operable}.
$P_i(j,t)\Delta$	Pr { at time t, i^{th} colour signal has failed }. Elapsed repair time lies in the interval $(j, j+\Delta)$
$P_{Yi}(j,z,t)\Delta$	Pr {at time t, i^{th} colour signal has failed while yellow colour signal is already failed}. Elapsed repair time for i^{th} colour signal lies in the interval $(j, j+\Delta)$ while for yellow colour signal it lies in the interval $(z, z+\Delta)$.
$P_{EW}(t)$	Pr {at time t, system is failed due to environmental reasons and is waiting for repair}.
$P_{ER}(m,t)\Delta$	Pr {at time t, system is failed due to environmental reasons and is ready for repair}. Elapsed repair time lies in the interval $(m, m+\Delta)$.
$R/G/Y$	Failure rate of red/green/yellow colour signal.
e_1, e_2	Failure rates due to environmental reasons.
$\mu_i(j)\Delta$	The first order probability that i^{th} failure will be repaired in the time interval $(j, j+\Delta)$, conditioned that it was not repaired up to time j.
W	Waiting rate for repair in case of environmental failure.
$\overline{P}(s)$	Laplace transform of function $P(t)$.
$S_i(j)$	$\mu_i(j)\exp.\left\{-\int \mu_i(j)dj\right\}, \forall\ i\ and\ j.$

FORMULATION OF MATHEMATICAL MODEL:

Probability considerations and limiting procedure yield the following set of difference-differential equations governing the behaviour of considered signal system:

$$\left(\frac{d}{dt}+R+G+Y+e_1\right)P_0(t)=\int_0^\infty P_R(x,t)\mu_R(x)dx+\int_0^\infty P_G(y,t)\mu_G(y)dy + \int_0^\infty P_Y(z,t)\mu_y(z)dz+\int_0^\infty P_{ER}(m,t)\mu_{ER}(m)dm \quad \ldots(1)$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\mu_R(x)\right]P_R(x,t)=0 \quad \ldots(2)$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+\mu_G(y)\right]P_G(y,t)=0 \quad \ldots(3)$$

$$\left[\frac{\partial}{\partial t}+W\right]P_{EW}(t)=e_1P_0(t)+e_2P_Y(t) \quad \ldots(4)$$

$$\left[\frac{\partial}{\partial m}+\frac{\partial}{\partial t}+\mu_{ER}(m)\right]P_{ER}(m,t)=0 \quad \ldots(5)$$

$$\left[\frac{\partial}{\partial z}+\frac{\partial}{\partial t}+R+G+e_2+\mu_Y(z)\right]P_Y(z,t)=\int_0^\infty P_{YR}(x,z,t)\mu_R(x)dx+\int_0^\infty P_{YG}(y,z,t)\mu_G(y)dy \quad \ldots(6)$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\mu_R(x)\right]P_{YR}(x,z,t)=0 \quad \ldots(7)$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+\mu_G(y)\right]P_{YG}(y,z,t)=0 \quad \ldots(8)$$

BOUNDARY CONDITIONS ARE:

$$P_R(0,t)=R\ P_0(t) \quad \ldots(9)$$

$$P_G(0,t)=G\ P_0(t) \quad \ldots(10)$$

$$P_Y(0,t)=Y\ P_0(t) \quad \ldots(11)$$

$$P_{YR}(0,z,t)=R\ P_Y(z,t) \quad \ldots(12)$$

$$P_{YG}(0,z,t)=G\ P_Y(z,t) \quad \ldots(13)$$

$$P_{ER}(0,t)=W\ P_{EW}(t) \quad \ldots(14)$$

INITIAL CONDITIONS ARE:

$$P_i(0)=\begin{cases}1, i=0\\ 0, i\neq 0\end{cases} \quad \ldots(15)$$

SOLUTION OF THE MODEL:

Taking Laplace transforms of equations (1) through (14) subjected to initial conditions (15), we obtain:

$$(s+R+G+Y+e_1)\overline{P}_0(s)=1+\int_0^\infty \overline{P}_R(x,s)\mu_R(x)dx+\int_0^\infty \overline{P}_G(y,s)\mu_G(y)dy$$
$$+\int_0^\infty \overline{P}_Y(z,s)\mu_y(z)dz+\int_0^\infty \overline{P}_{ER}(m,s)\mu_{ER}(m)dm \quad \ldots(16)$$

$$\left[\frac{\partial}{\partial x}+s+\mu_R(x)\right]\overline{P}_R(x,s)=0 \quad \ldots(17)$$

$$\left[\frac{\partial}{\partial y}+s+\mu_G(y)\right]\overline{P}_G(y,s)=0 \quad \ldots(18)$$

$$[s+W]\overline{P}_{EW}(s)=e_1\overline{P}_0(s)+e_2\overline{P}_Y(s) \quad \ldots(19)$$

$$\left[\frac{\partial}{\partial m}+s+\mu_{ER}(m)\right]\overline{P}_{ER}(m,s)=0 \quad \ldots(20)$$

$$\left[\frac{\partial}{\partial z}+s+R+G+e_2+\mu_Y(z)\right]\overline{P}_Y(z,s)=\int_0^\infty \overline{P}_{YR}(x,z,s)\mu_R(x)dx+\int_0^\infty \overline{P}_{YG}(y,z,s)\mu_G(y)dy \quad \ldots(21)$$

$$\left[\frac{\partial}{\partial x}+s+\mu_R(x)\right]\overline{P}_{YR}(x,z,s)=0 \quad \ldots(22)$$

$$\left[\frac{\partial}{\partial y}+s+\mu_G(y)\right]\overline{P}_{YG}(y,z,s)=0 \quad \ldots(23)$$

$$\overline{P}_R(0,s)=R\ \overline{P}_0(s) \quad \ldots(24)$$

$$\overline{P}_G(0,s)=G\ \overline{P}_0(s) \quad \ldots(25)$$

$$\overline{P}_Y(0,s)=Y\ \overline{P}_0(s) \quad \ldots(26)$$

$$\overline{P}_{YR}(0,z,s)=R\ \overline{P}_Y(z,s) \quad \ldots(27)$$

$$\overline{P}_{YG}(0,z,s)=G\ \overline{P}_Y(z,s) \quad \ldots(28)$$

$$\overline{P}_{ER}(0,s)=W\ \overline{P}_{EW}(s) \quad \ldots(29)$$

Now integrating equation (17) by using boundary condition (24) we get

$$\overline{P}_R(x,s)=R\overline{P}_0(s)\,e^{-sx-\int \mu_R(x)dx}$$

$$\Rightarrow \bar{P}_R(s) = R\bar{P}_0(s)\frac{1-\bar{S}_R(s)}{s} \quad \ldots(30)$$

or, $\bar{P}_R(s) = R\bar{P}_0(s)D_R(s)$ (say)

Similarly, solving (18) by using (25), we have:

$$\bar{P}_G(y,s) = G\bar{P}_0(s)\,e^{-sy-\int \mu_G(y)dy}$$
$$\Rightarrow \bar{P}_G(s) = G\bar{P}_0(s)D_G(s) \quad \ldots(31)$$

Again, integrating (20) subjected to (29), we get:

$$\bar{P}_{ER}(m,s) = W\,\bar{P}_{EW}(s)\,e^{-sm-\int \mu_{ER}(m)dm}$$
$$\Rightarrow \bar{P}_{ER}(s) = W\bar{P}_{EW}(s)D_{ER}(s) \quad \ldots(32)$$

Simplifying equation (19), we have :

$$\bar{P}_{EW}(s) = \frac{1}{s+W}\left[e_1\bar{P}_0(s) + e_2\bar{P}_Y(s)\right] \quad \ldots(33)$$

Now, integrating (22) by making use of (27), one may obtain:

$$\Rightarrow \bar{P}_{YR}(z,s) = R\bar{P}_Y(z,s)D_R(s) \quad \ldots(34)$$

Similarly, integrating (23) by using (28), we get:

$$\bar{P}_{YG}(y,z,s) = G\bar{P}_Y(z,s)\,e^{-sy-\int \mu_G(y)dy}$$
$$\Rightarrow \bar{P}_{YG}(z,s) = G\bar{P}_Y(z,s)D_G(s) \quad \ldots(35)$$

Now, equation (21) may be written as by using relevant relations:

$$\left[\frac{\partial}{\partial z} + s + R + G + e_2 + \mu_Y(z)\right]\bar{P}_Y(z,s) = R\bar{P}_Y(z,s)\bar{S}_R(s) + G\bar{P}_Y(z,s)\bar{S}_G(s)$$
$$\Rightarrow \left[\frac{\partial}{\partial z} + s + R + G + e_2 + \mu_Y(z) - R\bar{S}_R(s) - G\bar{S}_G(s)\right]\bar{P}_Y(z,s) = 0$$
$$\Rightarrow \bar{P}_Y(z,s) = Y\bar{P}_0(s)e^{-\left[s+R+G+e_2-R\bar{S}_R(s)-G\bar{S}_G(s)\right]Z-\int \mu_Y(z)dz}$$
$$\Rightarrow \bar{P}_Y(s) = Y\bar{P}_0(s)D_Y\left[s+R+G+e_2-R\bar{S}_R(s)-G\bar{S}_G(s)\right] \quad \ldots(36)$$
$$or, \bar{P}_Y(s) = Y\bar{P}_0(s)D_Y(A)$$

where, $A = s + R + G + e_2 - R\bar{S}_R(s) - G\bar{S}_G(s)$

By using (36), equations (34) and (35) gives:

$$\bar{P}_{YR}(s) = RD_R(s)Y\bar{P}_0(s)\bar{S}_Y(A) \quad \ldots(37)$$

and $\bar{P}_{YG}(s) = GD_G(s)Y\bar{P}_0(s)\bar{S}_Y(A)$ …(38)

In last, equation (16) gives on simplification:

$$\overline{P}_0(s)=\frac{1}{B(s)}$$

Thus, finally we have the following Laplace transforms of various transition-state probabilities:

$$\overline{P}_0(s)=\frac{1}{B(s)} \quad \ldots(39)$$

$$\overline{P}_R(s)=\frac{R}{B(s)}D_R(s) \quad \ldots(40)$$

$$\overline{P}_G(s)=\frac{G}{B(s)}D_G(s) \quad \ldots(41)$$

$$\overline{P}_{EW}(s)=\frac{1}{(s+W).B(s)}\left[e_1+e_2Y\,D_Y(A)\right] \quad \ldots(42)$$

$$\overline{P}_{ER}(s)=\frac{W}{(s+W).B(s)}\left[e_1+Ye_2\,D_Y(A)\right]D_{ER}(s) \quad \ldots(43)$$

$$\overline{P}_Y(s)=\frac{Y}{B(s)}D_Y(A) \quad \ldots(44)$$

$$\overline{P}_{YR}(s)=\frac{RYD_R(s)}{B(s)}\overline{S}_Y(A) \quad \ldots(45)$$

and $\overline{P}_{YG}(s)=\frac{GYD_G(s)}{B(s)}\overline{S}_Y(A)$ …(46)

where, $A=s+R+G+e_2-R\overline{S}_R(s)-G\overline{S}_G(s)$ …(47)

$$B(s)=s+R+G+Y+e_1-R\overline{S}_R(s)-G\overline{S}_G(s)-Y\overline{S}_Y(A)$$
$$-\frac{W}{s+W}\left[e_1+Ye_2\,D_Y(A)\right]\overline{S}_{ER}(s) \quad \ldots(48)$$

It is interesting to note that

Sum of equations (39) through (46) = $\frac{1}{s}$ …(49)

ERGODIC BEHAVIOUR OF THE SYSTEM:

By using final value theorem in Laplace transform, viz., $\lim_{t\to\infty} P(t)=\lim_{s\to 0} s\,\overline{P}(s)=P$ (say), provided the limit on L.H.S. exists, one can obtain the ergodic behaviour of the system from equations (39) through (46) as follows:

$$P_0=\frac{1}{\mathrm{B}'(0)} \quad \ldots(50)$$

$$P_R=\frac{\mathrm{R}}{\mathrm{B}'(0)}M_R \quad \ldots(51)$$

$$P_G=\frac{\mathrm{G}}{\mathrm{B}'(0)}M_G \quad \ldots(52)$$

$$P_{EW}=\frac{1}{WB'(0)}\left[e_1+e_2Y\,D_Y(e_2)\right] \quad \ldots(53)$$

$$P_{ER}=\frac{1}{B'(0)}\left[e_1+e_2 Y\, D_Y(e_2)\right]M_{ER} \qquad \ldots(54)$$

$$P_Y=\frac{Y}{B'(0)}D_Y(e_2) \qquad \ldots(55)$$

$$P_{YR}=\frac{RYM_R}{B'(0)}\overline{S}_Y(e_2) \qquad \ldots(56)$$

$$P_{YG}=\frac{GYM_G}{B'(0)}\overline{S}_Y(e_2) \qquad \ldots(57)$$

where, $M_i=-\overline{S}'_i(0), \forall i$

and $B'(0)=\left[\frac{d}{ds}B(s)\right]_{s=0}$

A PARTICULAR CASE:

WHEN ALL REPAIRS FOLLOW EXPONENTIAL TIME DISTRIBUTION :

In this case, we can obtain the Laplace transforms of various transition state probabilities from equations (39) through (46) by substituting $\overline{S}_i(j)=\frac{\mu_i}{j+\mu_i}, \forall\ i\ and\ j$ as follows:

$$\overline{P}_0(s)=\frac{1}{C(s)} \qquad \ldots(58)$$

$$\overline{P}_R(s)=\frac{R}{C(s)}.\frac{1}{s+\mu_R} \qquad \ldots(59)$$

$$\overline{P}_G(s)=\frac{G}{C(s)}.\frac{1}{s+\mu_G} \qquad \ldots(60)$$

$$\overline{P}_{EW}(s)=\frac{1}{(s+w)C(s)}\left[e_1+\frac{e_2Y}{\mu_Y+e_2+s+\frac{sR}{s+\mu_R}+\frac{sG}{s+\mu_G}}\right] \qquad \ldots(61)$$

$$\overline{P}_{ER}(s)=\frac{W}{(s+w)C(s)}\left[e_1+\frac{e_2Y}{\mu_Y+e_2+s+\frac{sR}{s+\mu_R}+\frac{sG}{s+\mu_G}}\right]\frac{1}{s+\mu_{ER}} \qquad \ldots(62)$$

$$\overline{P}_Y(s)=\frac{Y}{C(s)}\frac{1}{\mu_Y+e_2+s+\frac{sR}{s+\mu_R}+\frac{sG}{s+\mu_G}} \qquad \ldots(63)$$

$$\overline{P}_{YR}(s)=\frac{RY}{C(s)(s+\mu_R)}\frac{\mu_Y}{\mu_Y+e_2+s+\frac{sR}{s+\mu_R}+\frac{sG}{s+\mu_G}} \qquad \ldots(64)$$

and $\bar{P}_{YG}(s)=\dfrac{GY}{C(s)(s+\mu_G)}\dfrac{\mu_Y}{\mu_Y+e_2+s+\dfrac{sR}{s+\mu_R}+\dfrac{sG}{s+\mu_G}}$ …(65)

where, $C(s)=s\left[1+\dfrac{R}{s+\mu_R}+\dfrac{G}{s+\mu_G}\right]+Y+e_1-\dfrac{Y\mu_Y}{\mu_Y+e_2+s+\dfrac{sR}{s+\mu_R}+\dfrac{sG}{s+\mu_G}}$

$$-\frac{W}{s+W}\left[e_1+\frac{e_2Y}{\mu_Y+e_2+s+\dfrac{sR}{s+\mu_R}+\dfrac{sG}{s+\mu_G}}\right]\frac{\mu_{ER}}{s+\mu_{ER}} \quad \text{…(66)}$$

AVAILABILITY OF CONSIDERED SYSTEM:
We have

$$\bar{P}_{up}(s)=\frac{1}{s+R+G+Y+e_1}\left[1+\frac{Y}{s+R+G+e_2}\right]$$

On taking inverse Laplace transform, we obtain:

$$P_{up}(t)=(1+E)\exp.\{-(R+G+Y+e_1)t\}-E\exp.\{-(R+G+e_2)t\} \quad \text{…(67)}$$

where, $E=\dfrac{Y}{e_2-Y-e_1}$ …(68)

It is worth noticing that $P_{up}(0)=1$

Also, $P_{down}(t)=1-P_{up}(t)$ …(69)

PROFIT FUNCTION OF THE SYSTEM:
We know that the profit function is given by

$$G(t)=C_1\int_0^t P_{up}(t)dt-C_2t$$

where, C_1 and C_2 are revenue per unit time and repair cost per unit time, respectively. So here,

$$G(t)=C_1\left[\frac{1+E}{R+G+Y+e_1}\left\{1-e^{-(R+G+Y+e_1)t}\right\}-\frac{E}{R+G+e_2}\left\{1-e^{-(R+G+e_2)t}\right\}\right]-C_2t \quad \text{…(70)}$$

where, E has mentioned earlier.

NUMERICAL COMPUTATION:
For a numerical computation, let us consider the values:
$R=0.04,\ G=0.03,\ Y=0.01,\ e_1=0.001,\ e_2=0.002,\ C_1=Rs.7.00,\ C_2=Rs.2.00$ and t = 0,1,2,-----. By using these values in equations (67) and (70), one can compute the table- (1) and (2).

Table-1

t	P_{up} (t)
0	1
1	0.93145
2	0.86760
3	0.80812
4	0.75270
5	0.70108
6	0.65300
7	0.60820
8	0.56648
9	0.52761
10	0.49140

Table-2

t	G(t)
0	0
1	05.75726
3	13.9139
4	17.3745
5	19.4606
6	23.1979
7	25.6103
8	27.7201
9	28.5477
10	30.1128

RESULTS AND DISCUSSION:

In this study, the author deals with the transit traffic signal system for evaluation of its ability measures. Failures due to environmental reasons have also taken into account to obtain better results. Laplace transform of various transition state probabilities, shown in fig-1, have been obtained. Availability function and profit function for considered system have computed. Long run transition state probabilities and a particular case, when all repairs follow exponential time distribution, have been also determined to improve practical utility of the model. A numerical computation has mentioned to highlight important results of the study. By using this numerical computation, One can compute the table-1 and 2.

REFERENCES:

1. Gopalan, M.N.; Nagarwalla, H.E.: “Cost Benefits Analysis of a One-Server Two-unit Cold Standby System with Repair and Preventive Maintenances", Microelectronic Reliab. , Vol. 25, pp 267-269, 2013.
2. Gutierrez, P.H.; Aguirre, T. V.; Christen, J. A.: "A Practical Method for Obtaining Prior Distributions in Reliability”, IEEE TR. on Reliability, Vol. 54, issue - 2, pp 262 – 269, 2006.
3. Kuo, W; Prasad, V.R.; Tillman, F.A. and Hwang, C.L: "Fundamental and Applications of Reliability Optimization”, Cambridge University Press, Cambridge, 2013.
4. Linkov, L.N.: "Asymptotic Statistical Methods for Stochastic Processes", American Mathematical Society, 2013.

5. Pandey, D.; Jacob Mendus: "Cost Analysis, Availability and MTTF of a 3- State Standby Complex System under Common Cause and Human Failures", Microelectronic Reliab. , Vol.35, pp 91 -95, 2011.
6. Singh, J.; Pandey, P.C.; Kumar, D.: "Designing for Repairable Operation of Urea Synthesis in the Fertilizer Industry", Microelectronic Reliab. , Vol.309 (6), pp 1021 – 1024, 2010.
7. Tian, Z.; Yam, R. C. M.; Zuo, M. J.; Huang, H.Z.: "Reliability Bounds for Multi-State k-out-of- n Systems", IEEE TR. on Reliability, Vol.57 (1), pp 53-58, 2008.
8. Wang, K.H.; Hsu, L.Y.: "Cost Analysis of the Machine Repair Problem with Reliability Non-Reliable Service Stations", Microelectronic Reliab. , Vol. 35 (6), pp 923-934, 1996.
9. Way Kuo: "Compatibility and Simplicity: The Fundamentals of Reliability", IEEE TR. on Reliability, Vol.56 (4), pp 585-586, 2008.
10. Wayn Nelson, J.Wiley and Sons, ―Applied life Data Analysis, Wiley Series in Probability and Statistics, February 2005.
11. Yu Liu Hong-Zhong Huang: "Optimal Replacement Policy for Multi-State System Under Imperfect Maintenance", IEEE Transaction on Reliability, Vol.59, Issue-3, pp 483-495, 2011.